Traffic-Adaptive Spectrum Leasing Between Primary and Secondary Networks

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***Abstract*—Spectrum leasing has been widely regarded as one of the most effective ways to improve the utilization of limited spectrum resources. In this paper, we propose a novel traffic-adaptive spectrum leasing (TASL) scheme by allowing secondary users (SUs) to lease part of licensed spectrum channels from primary users (PUs) temporarily for transmitting the dynamically generated secondary packets. As the time length of each leasing period is variable according to the dynamic generation of secondary packets, the proposed TASL can effectively satisfy the quality-of-service requirement of SUs and also benefit PUs with the financial payoff provided by SUs. By establishing a three-dimensional continuous Markov chain for the proposed TASL, we formulate the average utilities of PUs and SUs in terms of the expected buffering time of primary and secondary packets, as well as the expected transmission throughput of PUs and SUs. Moreover, to coordinate the interests of PUs and SUs in a noncooperative manner, we also propose a Stackelberg game model for PUs and SUs to negotiate various spectrum leasing parameters and further apply two specific rules to guarantee the existence of a unique equilibrium solution. Numerical simulation shows that, compared with those existing spectrum leasing schemes that preset a fixed-time length for leasing periods, the proposed TASL can effectively improve the utilization of the leased channels, increase the average utilities of both PUs and SUs, and be more suitable for newly emerging applications that are sensitive to packet transmission delay and buffering overhead.**

***Index Terms*—Spectrum leasing, traffic adaptive, Markov chain, quality of service, Stackelberg game.**

I. Introduction

ACCOMPANYING with the fast increasing of new applications and traffic demand, the under-utilization of limited spectrum resources is becoming a performance bottleneck of wireless communications [1]. By allowing primary users (PUs) to lease unused licensed spectrum to secondary users (SUs) temporarily for receiving financial payoff or a certain type of services, e.g., cooperative relay, spectrum leasing has been widely regarded as one of the most effective ways to improve the efficiency of spectrum utilization ([2], [3]). In cognitive radio applications, spectrum leasing can also save the time and energy consumption for SUs to detect the agile spectrum holes [4] and help PUs to reduce the harmful interference from those SUs that fail to detect the activity of PUs.

The existing spectrum leasing schemes can be divided into two major categories, i.e., the *underlay* and *overlay* schemes. In underlay schemes [5]–[7], PUs and SUs utilize a common set of licensed spectrum channels simultaneously under the condition that the total interference from SUs to PUs is below a prescribed threshold, namely *interference temperature* (IT). This threshold, however, limits the transmission power of SUs, especially when the number of SUs is large or SUs and PUs locate closely, and makes it difficult for SUs to satisfy their basic requirements on the quality of service (QoS) [8].

Thus more research works focus on the design of overlay spectrum leasing schemes, which allow SUs to lease licensed channels without limiting their transmission powers. For example, the cooperative relay schemes [9]–[17] allow multiple SUs to lease licensed channels from one PU for transmitting secondary packets (SPs) temporarily at the cost of providing cooperative relay service for primary packets (PPs). Meanwhile, the literatures [18]–[27] consider the spectrum leasing from multiple PUs to multiple SUs for receiving various types of services such as cooperative relay, data offloading, and interference mitigation. These schemes incorporate various communication techniques, e.g., superposition coding [10]–[12], beamforming [16], and successive interference cancellation ([14], [23]), for improving the transmission utilities, sum rate, or energy efficiency of PUs and SUs and derive the optimal solution for relay selection, resource allocation, or pricing based on various game-theoretical frameworks ([2], [14]–[16], [20], [21], [24]–[27]) or multi-objective optimizations ([9], [10], [18], [19]). In these schemes, as SUs should always spend an non-negligible time length to implement its services for PUs, the infinite time of the leased spectrum channels can be divided into two types of non-overlapping time intervals, namely *leasing* and *buffering periods*. In each buffering period, SUs can implement its services for PUs at the leased channels and buffer newly generated SPs concurrently, while, in each leasing period, SUs can transmit its buffering and newly generated SPs at the leased channels. Regardless of their application scenarios, all these works show that both PUs and SUs can benefit from spectrum leasing and hence be motivated to take part in it.

However, these works also share a common shortcoming, i.e., as the PUs and SUs always negotiate a fixed time length for all leasing periods, this preset time length cannot adapt to the real-time transmission demand of SUs in most time, which normally

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varies randomly over time ([3], [28]). In particular, if the preset leasing period is longer than the required transmission time of SPs, then the leased channels will be wasted after these SPs have been transmitted; else, if the former is shorter than the latter, then part of SPs generated in a leasing period will not be transmitted in this period and hence have to be buffered until the next leasing period, which, nevertheless, degrades the QoS of SP transmission. Recently [28] claimed itself as the first work to implement spectrum leasing in a dynamic manner for serving the stochastic demands of SUs. However, it mainly changes the leasing prices over time dynamically based on SUs' stochastic demands and still sets a fixed time length for the leasing periods of those SUs with a certain level of traffic demand. Thus, as concluded by the latest survey [3], it is still a research challenge to develop a traffic-adaptive spectrum leasing scheme considering the real-time variations of traffic demands.

Moreover, as all spectrum leasing schemes inevitably require PUs or SUs to buffer their generated packets temporarily, the more PPs or SPs that the PUs or SUs are buffering, respectively, the longer the average delay experienced by PPs or SPs, and the worse the transmission performance of PUs or SUs. In practice, many newly emerging applications, e.g., mobile games and e-health, are also very sensitive to the cost of packet buffering. Thus it is desirable for a spectrum leasing scheme to take the buffering costs of PPs and SPs into consideration. However, this issue has been largely ignored in the existing spectrum leasing schemes, e.g., [12]–[25], [28].

In view of this, the present paper proposes a novel traffic-adaptive spectrum leasing (TASL) scheme by allowing SUs to lease a certain number of licensed spectrum channels from PUs for transmitting the dynamically generated SPs. By setting a variable time length for each leasing period according to the real-time generation and transmission of SPs, the proposed TASL can effectively satisfy the QoS requirement of SUs and, meanwhile, benefit PUs with financial payment, which can also be interpreted as the cooperative service provided by SUs. To analyze the transmission performance and buffering cost of both PUs and SUs in the TASL, we then establish a three-dimensional continuous Markov chain model to describe the state transition of TASL, derive the explicit expressions of the expected buffering time and transmission throughput for both PUs and SUs, and formulate their long-term average utilities in spectrum leasing. Moreover, to coordinate the interests of PUs and SUs in a non-cooperative manner, we also propose a Stackelberg game model for them to negotiate various spectrum leasing parameters subject to their basic QoS requirements and apply two specific rules to guarantee the existence of a unique equilibrium solution for the TASL.

The key contributions of this paper are summarized as follows:

- Compared with the existing spectrum leasing schemes ([12]–[25], [28]), which preset a fixed time length for leasing periods and ignore the buffering costs of PPs and SPs, the proposed TASL stipulates the time length of each leasing period to be variable according to the real-time generation and transmission of SPs, which guarantees the exact matching of each leasing period with the stochastic demand of SUs, and can achieve a unique equilibrium solution of spectrum leasing by considering the expected transmission throughput and buffering time of PUs and SUs concurrently.
- To derive the equilibrium solution for the proposed TASL, this paper adopts the solution technique of joint queueing analysis and game-theoretical modeling, which characterises the stochastic traffic behavior of PUs and SUs by a three-dimensional continuous Markov chain and develops a Stackelberg game model based on the average utilities of PUs and SUs. To our best knowledge, this technique has rarely been applied in non-cooperative spectrum leasing for PUs and SUs to take their long-term transmission benefit and buffering cost into consideration.
- Numerical simulation shows that, compared with the so-called *partially traffic-adaptive spectrum leasing* (*PTASL*) scheme that generalizes the existing spectrum leasing schemes, e.g., [9]–[11], [14]–[20], and [22], and predetermines a fixed time length for leasing periods, the proposed TASL can effectively improve the utilization of the leased channels, increase the average utilities of PUs and SUs in spectrum leasing, and be more suitable for newly emerging applications that are sensitive to packet transmission delay and buffering overhead.

TABLE I
SUMMARY FOR KEY NOTATIONS IN THIS PAPER

Notation	Explanation
N	Total no. of spectrum channels licensed to PN
N_S	No. of licensed channels leased by SN
N_P	No. of licensed channels reserved by PN
n_S or n_P	Maximal no. of SPs or PPs buffered by SN or PN
n	Maximal no. of buffering SPs accumulated by SN in a buffering period of TASL
λ_S or λ_P	Ave. generation rate of SPs or PPs
μ_S or μ_P	Ave. transmission rate of SPs or PPs
T_L or T_B	Time length of a leasing or buffering period
T_{pd} or α	$T_{pd} = T_L + T_B$ or $\alpha = \frac{T_L}{T_{pd}}$
c_S or c_P	Cost for buffering one SP or PP in a unit time
e_S or e_P	Revenue for transmitting one SP or PP
K	Fixed charge for PN to clear a licensed channel
p	Price for leasing a licensed channel per unit time
U_{SN} or U_{PN}	Ave. utility of SN or PN
$N_{P,L}^{(0)}$ or $N_{P,B}^{(0)}$	Ave. no. of initial PPs in a leasing/buffering period
$B_{S,L}$ or $B_{S,B}$	Buffering time of SN in a leasing/buffering period
$B_{P,L}$ or $B_{P,B}$	Buffering time of PN in a leasing/buffering period
$R_{S,L}$	Throughput of SN in a leasing period
$R_{P,L}$ or $R_{P,B}$	Throughput of PN in a leasing/buffering period

Table I summarizes the key notations adopted in this paper and the remaining of this paper is organized as follows. Section II describes the system model and presents the TASL, based on which Section III develops a three-dimensional Markov chain model for the TASL. Following this model, Section IV formulates the average utilities of SUs and PUs in the TASL and Section V develops a Stackelberg game model for SUs and PUs to negotiate spectrum leasing parameters. Numerical simulation in Section VI then validates the theoretical analysis and compares the performance of TASL and PTASL. Finally, Section VII concludes the main contribution of this paper.

II. System Model and the Proposed TASL Scheme

Consider a wireless system consisting of one primary network (PN) and one secondary network (SN). In this system, the SN, composed of one secondary access point (SAP) and multiple SUs, is a single-hop ad hoc network lack of legal spectrum resources, while the PN, composed of one primary base station (PBS) and multiple PUs, has totally $N(\geq 1)$ disjoint

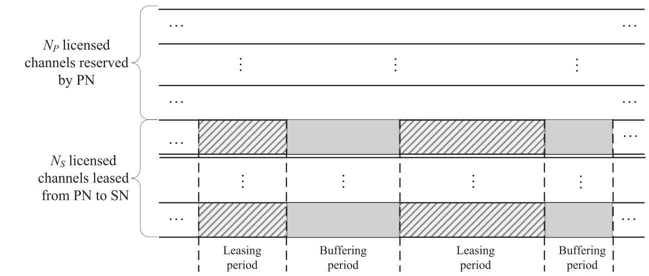


Fig. 1. In the proposed traffic-adaptive spectrum leasing (TASL) scheme, the infinite time of the N_S licensed channels leased from the PN to the SN are synchronously divided into leasing and buffering periods with variable time lengths, which can fully adapt to the real-time secondary traffic.

licensed spectrum channels with uniform bandwidth for primary transmission. To satisfy its transmission demand, the SN has to pay a price to rent a certain number of licensed channels temporarily from the PN for secondary transmission. Meanwhile, to maximize its benefit without affecting the QoS requirement of primary transmission, the PN can lease part of the N licensed channels to the SN for financial payoff or cooperative service.

Once the SN leases a certain number of licensed channels from the PN, the SAP will regulate all SUs to sequentially transmit their buffered SPs over the leased channels according to an appropriate order, which may be determined by traffic type (e.g., real-time or non-real-time), traffic emergency, user priority, etc. Whenever a SU generates a SP, it has to first buffer this packet and then report it to the SAP via a narrow-band dedicated control channel, which is free of interference. Only after the SU obtains an approval from the SAP, it will begin to transmit its buffering SPs over the leased channel specified by the SAP. Thus there exists no transmission collision within the SN. As the SAP normally assigns at most one leased channel for each SU, multiple SUs can transmit their SPs at different leased channels at the same time. Meanwhile, as the PBS coordinates the packet transmission among PUs in the same way as the SAP does, there also exists no transmission collision inside the PN.

Because the SN and PN have different interests, the key problem is how to form a mutually beneficial agreement for spectrum leasing between them. For this purpose, the present paper proposes a traffic-adaptive spectrum leasing (TASL) scheme for the SN to lease a certain number of licensed channels from the PN according to the real-time traffic demands of SUs. In this scheme, if the SN successfully leases $N_S \in [1, N]$ licensed channels, the infinite time of these channels will be synchronously divided into *leasing* and *buffering* periods, which alternatively appear in the time axis as shown in Fig. 1. A leasing period begins when the SN buffers exactly $n(\geq 1)$ SPs and ends when the buffers of all SUs become empty, while the time interval between any two adjacent leasing periods is a buffering period. The SN can utilize the N_S leased channels for transmitting SPs in leasing periods and only accumulates buffering SPs in buffering periods, while the PN can always utilize the $N_P (= N - N_S)$ unleased channels for transmitting PPs at any time and also occupy the N_S leased channels for the same purpose in buffering periods. Thus the concepts of leasing and buffering periods only apply to the leased channels and have no relationship with the PP transmission at the unleased channels. To guarantee the effective execution of TASL in the long run, assume that both SN and PN are always willing to exchange their transmission and buffering parameters honestly.

The packet generation process of SN or PN follows the Poisson distribution with the parameter λ_S or λ_P, respectively, and the transmission process of SPs or PPs over a licensed channel follows an exponential distribution with the parameter μ_S or μ_P. As these four traffic parameters can be estimated by various existing approaches, e.g., [29] and [30], the expected time length of a buffering period can be calculated as

$$E[T_B] = \frac{n}{\lambda_S}, \tag{1}$$

while, according to [32], the expected time length of a leasing period can be expressed as

$$E[T_L] = \sum_{j=1}^{\infty} \rho_j + \sum_{m=1}^{n-1} \left[\prod_{k=1}^{m} \frac{\mu_k}{\lambda_S} \sum_{j=m+1}^{\infty} \rho_j \right], \tag{2}$$

where $\rho_j = \frac{\lambda_S^{j-1}}{\mu_1 \mu_2 \ldots \mu_j}$ and μ_k is equal to $k\mu_S$ for $1 \leq k < N_S$ or $N_S \mu_S$ for $k \geq N_S$.

In order to meet the QoS requirement of secondary transmissions, the selection for the number N_S of leased channels should avoid the overflow of SU buffers in the long run. Thus the expected total transmission time of SN in one alternation of buffering and leasing periods, i.e., $N_S E[T_L]$, should be no smaller than the expected total transmission time of all SPs newly generated in the same time, i.e., $\frac{\lambda_S E[T_{pd}]}{\mu_S}$, where $E[T_{pd}] = E[T_L] + E[T_B]$. That is,

$$N_S \geq \left\lceil \frac{\lambda_S E[T_{pd}]}{\mu_S E[T_L]} \right\rceil. \tag{3}$$

Similarly, to guarantee the QoS requirement of primary transmissions, the expected total transmission time of PN in one alternation of buffering and leasing periods, i.e., $N_P E[T_L] + N E[T_B]$, should be no smaller than the expected total transmission time of all PPs generated in the same time, i.e., $\frac{\lambda_P E[T_{pd}]}{\mu_P}$. That is,

$$N_S \leq \left\lfloor \frac{N E[T_{pd}]}{E[T_L]} - \frac{\lambda_P E[T_{pd}]}{\mu_P E[T_L]} \right\rfloor. \tag{4}$$

Thus the target of SN or PN in the proposed TASL is to maximize its utility in spectrum leasing subject to the constraint (3) or (4), respectively. To evaluate these utilities, the next section will investigate the state transition of SN and PN in the TASL.

III. Three-Dimensional Markov Model for TASL

To describe the long-term probabilistic behavior of SN and PN in the proposed TASL, this section establishes a continuous finite-state Markov chain $\{CH_t, SP_t, PP_t\}$, where $CH_t \in \{N_P, N\}$ denotes the number of licensed channels available for PP transmission at any time $t \geq 0$, SP_t the total number of SPs being buffered and transmitted by the SN at the time t, and PP_t the total number of PPs being buffered and transmitted by the PN at the time t. In particular, when $CH_t = N_P$ or N, the Markov chain will be said in a *leasing* or *buffering* state, respectively.

Note that the range of SP_t depends on whether the Markov chain is in a leasing or buffering state at the time t. That is, the range of SP_t is $[1, n_S]$ in a leasing state or $[0, n-1]$ in a buffering state, where n_S denotes the maximal number of SPs that the SN can buffer physically and n is the maximal number of buffering SPs that the TASL allows the SN to accumulate in a

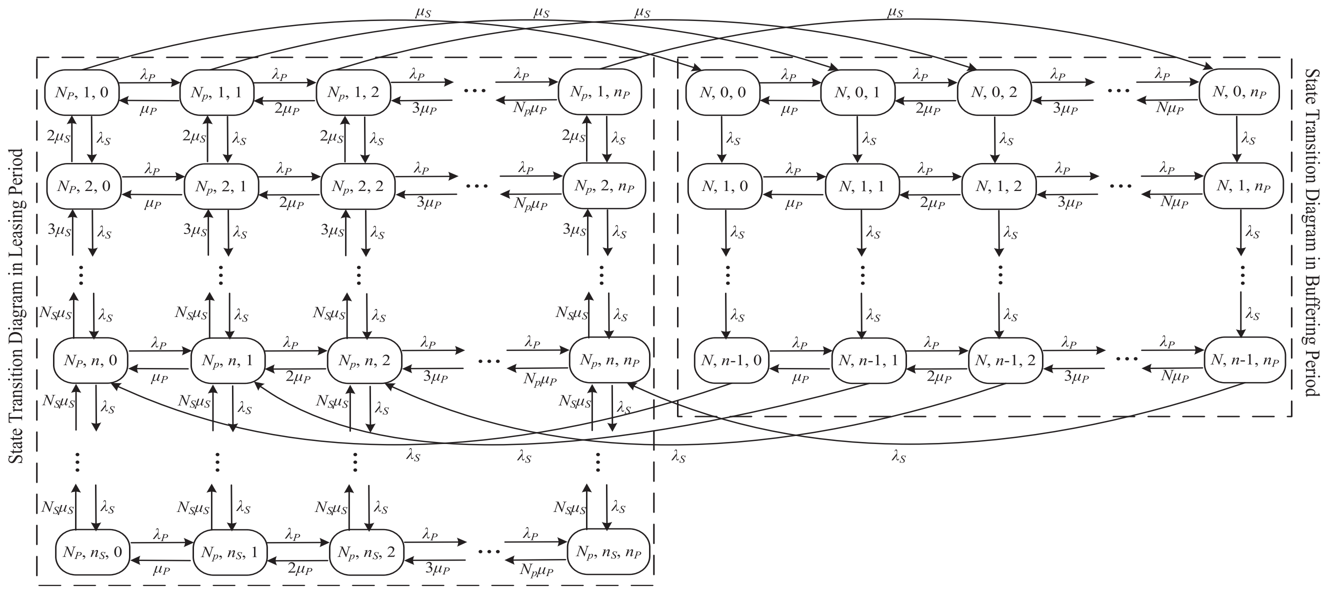


Fig. 2. State transition diagram of the Markov chain $\{CH_t, SP_t, PP_t\}$.

buffering period. On the other hand, the range of PP_t is $[0, n_P]$ for both leasing and buffering states, where n_P is the maximal number of PPs that the PN can buffer physically. Accordingly, the Markov chain $\{CH_t, SP_t, PP_t\}$ has a total number of $(n_S + n)(n_P + 1)$ states. Because nowadays random access memory (RAM) becomes fairly cheap [31], it is reasonable to assume that

(*) $n_S \gg n, n_S \gg N_S$, and $n_P \gg N$.

Following this assumption, Fig. 2 illustrates the state transition diagram of the proposed Markov chain. In this diagram, each horizontal (*resp*. vertical) arrow from the leasing state (N_P, i, j) (*resp*. (N_P, k, h)) to the leasing state $(N_P, i, j+1)$ (*resp*. $(N_P, k+1, h)$), where $i \in [1, n_S]$ (*resp*. $k \in [1, n_S - 1]$) and $j \in [0, n_P - 1]$ (*resp*. $h \in [0, n_P]$), represents the state transition induced by the generation of a new PP (*resp*. SP) at the time when exactly j (*resp*. k) PPs (*resp*. SPs) are being buffered and transmitted, while that from the leasing state $(N_P, i, j+1)$ (*resp*. $(N_P, k+1, h)$) to the leasing state (N_P, i, j) (*resp*. (N_P, k, h)) denotes the state transition induced by the successful transmission of a PP (*resp*. SP) at the time when exactly $j+1$ (*resp*. $k+1$) PPs (*resp*. SPs) are being buffered and transmitted.

Similar as the state transition among leasing states, Fig. 2 also shows the state transition from the buffering state (N, i, j) (*resp*. (N, k, h)) to the buffering state $(N, i, j+1)$ (*resp*. $(N, k+1, h)$), where $i \in [0, n-1]$ (*resp*. $k \in [0, n-2]$) and $j \in [0, n_P - 1]$ (*resp*. $h \in [0, n_P]$), which is induced by the generation of a new PP (*resp*. SP) at the time when exactly j PPs (*resp*. k SPs) are being buffered and transmitted, as well as that from the buffering state $(N, i, j+1)$ to the buffering state (N, i, j) induced by the successful transmission of a PP at the time when exactly $j+1$ PPs are being buffered and transmitted. Since the SN cannot transmit any SP in a buffering period, there exists no state transition among buffering states caused by the successful transmission of the SP.

Moreover, each curvy arrow in Fig. 2 from the leasing state $(N_P, 1, l)$ to the buffering state $(N, 0, l)$, where $l \in [0, n_P]$, represents the state transition induced by the successful transmission of the final SP in a leasing period, while that from the buffering state $(N, n-1, l)$ to the leasing state (N_P, n, l) labels the state transition induced by the generation of the n^{th} SP in a buffering period. These are the two possible state transitions between buffering and leasing periods.

Based on Fig. 2, the remaining of this section will first derive the state transition matrix of the Markov chain and then calculate the stationary distribution of all possible states of the Markov chain.

A. State Transition Matrix of the Markov Chain

By Fig. 2, we formulate the state transition matrix $\mathbf{Q}$ of the Markov chain $\{CH_t, SP_t, PP_t\}$ as an $(n + n_S) \times (n + n_S)$ matrix in (5) shown at the top of next page, inside which each element is an $(n_P + 1) \times (n_P + 1)$ submatrix of state transition rates. The notation for each submatrix has a subscript i, which implies that all state transitions labeled by this submatrix departing from or arriving at a state with i SPs, and a superscript of N_P or N, which implies that the state with i SPs is a leasing or buffering state, respectively. Each submatrix at the column $u \in [1, n_S]$ of the matrix $\mathbf{Q}$ represents those transitions departing from or arriving at a leasing state with u SPs, while that at the column $v \in [n_S + 1, n + n_S]$ of the same matrix labels those transitions departing from or arriving at a buffering state with $(v - n_S - 1)$ SPs. Below we describe various submatrices involved in (5):

- Each submatrix $\mathbf{E}_i^{N_P}$ for $i \in [1, n_S]$ can be expressed as the summation of two $(n_P + 1) \times (n_P + 1)$ submatrices $\mathbf{F}_i^{N_P}$ and $\mathbf{G}_i^{N_P}$, while each submatrix $\mathbf{E}_i^{N}$ for $i \in [0, n-1]$ can be expressed as the summation of two $(n_P + 1) \times (n_P + 1)$ submatrices $\mathbf{F}_i^{N}$ and $\mathbf{G}_i^{N}$.
- Each $\mathbf{F}_i^{N_P}$, where $i \in [1, n_S]$, represents the transition departing from a leasing state with i SPs due to the generation of a new PP or SP as well as the successful transmission of a PP or SP, while each $\mathbf{F}_i^{N}$, where $i \in [0, n-1]$, denotes the transition departing from a buffering state with i SPs due to the generation of a new PP or SP as well as the successful transmission of a PP[1].

[1]Note that no SP can be transmitted in a buffering period.

$$\mathbf{Q} = \begin{pmatrix}
\mathbf{E}_1^{N_P} & \mathbf{B}_2^{N_P} & & & & & & & \mathbf{H}_0^{N} & & & & & \\
\mathbf{D}_1^{N_P} & \mathbf{E}_2^{N_P} & \mathbf{B}_3^{N_P} & & & & & & & & & & & \\
 & \mathbf{D}_2^{N_P} & \mathbf{E}_3^{N_P} & \ddots & & & & & & & & & & \\
 & & \ddots & \ddots & \mathbf{B}_n^{N_P} & & & & & & & & & \\
 & & & \ddots & \mathbf{E}_n^{N_P} & \ddots & & & & & & & & \\
 & & & & \mathbf{D}_n^{N_P} & \ddots & \mathbf{B}_{n_S-1}^{N_P} & & & & & & & \\
 & & & & & \ddots & \mathbf{E}_{n_S-1}^{N_P} & \mathbf{B}_{n_S}^{N_P} & & & & & & \\
 & & & & & & \mathbf{D}_{n_S-1}^{N_P} & \mathbf{E}_{n_S}^{N_P} & & & & & & \\
 & & & & & & & & \mathbf{E}_0^{N} & \mathbf{B}_1^{N} & & & & \\
 & & & & & & & & & \mathbf{E}_1^{N} & \mathbf{B}_2^{N} & & & \\
 & & & & & & & & & & \mathbf{E}_2^{N} & \mathbf{B}_3^{N} & & \\
 & & & & & & & & & & & \mathbf{E}_3^{N} & \ddots & \\
 & & & & & & & & & & & & \ddots & \mathbf{B}_{n-1}^{N} \\
 & & & & \mathbf{H}_n^{N_P} & & & & & & & & & \mathbf{E}_{n-1}^{N}
\end{pmatrix}, \quad (5)$$

- Each $\mathbf{G}_i^{N_P}$ (*resp.* $\mathbf{G}_i^{N}$), where $i \in [1, n_S]$ (*resp.* $i \in [0, n-1]$), represents the transition arriving at a leasing (*resp.* buffering) state with i SPs due to the generation of a new PP or the successful transmission of a PP.
- Each $\mathbf{B}_i^{N_P}$ (*resp.* $\mathbf{B}_i^{N}$), where $i \in [2, n_S]$ (*resp.* $i \in [1, n-1]$), denotes the transition arriving at a leasing (*resp.* buffering) state with i SPs due to the generation of a new SP, while each $\mathbf{D}_i^{N_P}$, where $i \in [1, n_S - 1]$, denotes the transition arriving at a leasing state with i SPs due to the successful transmission of a SP.
- The $\mathbf{H}_n^{N_P}$ represents the transition from a buffering state with $n-1$ SPs to a leasing state with n SPs, while $\mathbf{H}_0^{N}$ denotes the transition from a leasing state with only one SP to a buffering state with zero SP.

In Fig. 2, $\mathbf{F}_i^{N_P}$ or $\mathbf{F}_i^{N}$ represents all horizontal and vertical arrows originating from the row-i leasing or buffering states, respectively, $\mathbf{G}_i^{N_P}$, $\mathbf{B}_i^{N_P}$ and $\mathbf{D}_i^{N_P}$ together represent all horizontal and vertical arrows terminating at the row-i leasing states, while $\mathbf{G}_i^{N}$ and $\mathbf{B}_i^{N}$ together represent all horizontal and vertical arrows terminating at the row-i buffering states. Moreover, for all $l \in [0, n_P]$, $\mathbf{H}_0^{N}$ represents all curvy arrows from the leasing states $(N_P, 1, l)$ to the buffering states $(N, 0, l)$ and $\mathbf{H}_n^{N_P}$ those from the buffering states $(N, n-1, l)$ to the leasing states (N_P, n, l).

1) Derivation of $\mathbf{E}_i^{N_P}$ and $\mathbf{E}_i^{N}$: Since $\mathbf{E}_i^{N_P} = \mathbf{F}_i^{N_P} + \mathbf{G}_i^{N_P}$ and $\mathbf{E}_i^{N} = \mathbf{F}_i^{N} + \mathbf{G}_i^{N}$, we only need to formulate $\mathbf{F}_i^{N_P}$, $\mathbf{G}_i^{N_P}$, $\mathbf{F}_i^{N}$, and $\mathbf{G}_i^{N}$. In a leasing period, each submatrix $\mathbf{F}_i^{N_P}$ for $1 \le i \le n_S$ can be expressed as $\mathrm{diag}(f_{i,0}^{N_P}, \ldots, f_{i,n_P}^{N_P})$, where, for each $0 \le j \le n_P$, $f_{i,j}^{N_P} = f_{i,j,SP}^{N_P} + f_{i,j,PP}^{N_P}$, $f_{i,j,PP}^{N_P}$ is the summation of the transition rate from the state $\{N_P, i, j\}$ to the state $\{N_P, i, j+1\}$ due to the generation of a new PP and that from the state $\{N_P, i, j\}$ to the state $\{N_P, i, j-1\}$ due to the successful transmission of a PP, and $f_{i,j,SP}^{N_P}$ is the summation of the transition rate from the state $\{N_P, i, j\}$ to the state $\{N_P, i+1, j\}$ due to the generation of a new SP and that from the state $\{N_P, i, j\}$ to the state $\{N_P, i-1, j\}$ due to the successful transmission of an SP. From Fig. 2, if $0 \le j \le N_P - 1$, then $f_{i,j,PP}^{N_p} = -\lambda_P - j\mu_P$; else, if $N_P \le j \le n_P - 1$, then $f_{i,j,PP}^{N_p} = -\lambda_P - N_P\mu_P$; else, if $j = n_P$, then $f_{i,j,PP}^{N_p} = -N_P\mu_P$. Moreover, if $0 \le i \le N_S - 1$, then $f_{i,j,SP}^{N_p} = -\lambda_S - i\mu_S$; else, if $N_S \le i \le n_S - 1$, then $f_{i,j,SP}^{N_p} = -\lambda_S - N_S\mu_S$; else, if $i = n_S$, then $f_{i,j,SP}^{N_p} = -N_S\mu_S$.

On the other hand, each submatrix $\mathbf{F}_i^{N}$ for $0 \le i \le n-1$ can be expressed as $\mathrm{diag}(f_{i,0}^{N}, \ldots, f_{i,n_P}^{N})$, where, for each $0 \le j \le n_P$, $f_{i,j}^{N} = f_{i,j,PP}^{N} + f_{i,j,SP}^{N}$, $f_{i,j,PP}^{N}$ is the summation of the transition rate from $\{N, i, j\}$ to $\{N, i, j+1\}$ due to the generation of a new PP and that from $\{N, i, j\}$ to $\{N, i, j-1\}$ due to the successful transmission of a PP, and $f_{i,j,SP}^{N}$ is only the transition rate from $\{N, i, j\}$ to $\{N, i+1, j\}$ due to the generation of a new SP. Because the generation of a new PP or the successful transmission of a PP in a buffering period follows the same transition rate as that in a leasing period, we have $f_{i,j,PP}^{N}$ has the similar definition as $f_{i,j,PP}^{N_P}$ and $f_{i,j,SP}^{N} = -\lambda_S$.

Moreover, as $\mathbf{G}_i^{N_P}$ is independent of the parameter i, the n_S submatrices $\mathbf{G}_1^{N_P}, \mathbf{G}_2^{N_P}, \ldots, \mathbf{G}_{n_S}^{N_P}$ are all equal to the following $(n_P+1) \times (n_P+1)$ matrix:

$$\mathbf{G} = \begin{pmatrix}
0 & g_{0,1}^{N_P} & & & & & \\
g_{1,0}^{N_P} & 0 & g_{1,2}^{N_P} & & & & \\
 & g_{2,1}^{N_P} & 0 & g_{2,3}^{N_P} & & & \\
 & & \ddots & \ddots & \ddots & & \\
 & & & g_{n_P-1,n_P-2}^{N_P} & 0 & g_{n_P-1,n_P}^{N_P} \\
 & & & & g_{n_P,n_P-1}^{N_P} & 0
\end{pmatrix}, \quad (6)$$

where, for each $0 \leq j \leq n_P - 1$, $g^{N_P}_{j,j+1}$ denotes the transition rate from $\{N_P, i, j\}$ to $\{N_P, i, j+1\}$ due to the generation of a new PP and $g^{N_P}_{j+1,j}$ denotes that from $\{N_P, i, j+1\}$ to $\{N_P, i, j\}$ due to the successful transmission of a PP. Similarly, the n submatrices $\mathbf{G}^N_0$, $\mathbf{G}^N_1$, ..., $\mathbf{G}^N_{n-1}$ are all equal to an $(n_P+1) \times (n_P+1)$ matrix, which is same as $\mathbf{G}$ except that each N_P in (6) should be replaced by N. From Fig. 2, we have $g^{N_P}_{j,j+1} = g^N_{j,j+1} = \lambda_P$, $g^{N_P}_{j+1,j}$ is equal to $(j+1)\mu_P$ for $0 \leq j \leq N_P - 1$ or $N_P \mu_P$ for $j \geq N_P$, and $g^N_{j+1,j}$ is equal to $(j+1)\mu_P$ for $0 \leq j \leq N-1$ or $N\mu_P$ for $j \geq N$.

2) Derivation for Other Submatrices in (5): Since $\mathbf{H}^{N_P}_n$, $\mathbf{B}^{N_P}_2 \sim \mathbf{B}^{N_P}_{n_S}$, and $\mathbf{B}^N_1 \sim \mathbf{B}^N_{n-1}$ represent the state transitions due to the generation of a new SP, we have $\mathbf{H}^{N_P}_n = \mathbf{B}^{N_P}_2 = \mathbf{B}^{N_P}_3 = \cdots = \mathbf{B}^{N_P}_{n_S} = \mathbf{B}^N_1 = \mathbf{B}^N_2 = \cdots = \mathbf{B}^N_{n-1} = \lambda_S \mathbf{I}$, where $\mathbf{I}$ denotes the $(n_P+1) \times (n_P+1)$ identity matrix. Since $\mathbf{H}^N_0$ and $\mathbf{D}^{N_P}_1 \sim \mathbf{D}^{N_P}_{n_S-1}$ represent the state transition due to the successful transmission of an SP, we have $\mathbf{H}^N_0 = \mu_S \mathbf{I}$ and $\mathbf{D}^{N_P}_i$ is equal to $(i+1)\mu_S \mathbf{I}$ for $1 \leq i \leq N_S - 1$ or $N_S \mu_S \mathbf{I}$ for $i \geq N_S$.

B. Stationary Distribution of the Markov Chain

As shown by Fig. 2, there always exists a non-zero transition probability between any two states of the finite-state Markov chain $\{CH_t, SP_t, PP_t\}$. By [33], this fact implies that the Markov chain has a unique stationary distribution. Denote by $p(N_P, j, k)$ the stationary probability for the leasing state (N_P, j, k), where $j \in [1, n_S]$ and $k \in [0, n_P]$, and by $p(N, l, k)$ the stationary probability for the buffering state (N, l, k), where $l \in [0, n-1]$. Thus the row vector for the stationary distribution of the Markov chain $\{CH_t, SP_t, PP_t\}$, i.e., $\pi = \{p(N_P, 1, 0), p(N_P, 1, 1), \ldots, p(N_P, n_S, n_P), p(N, 0, 0), p(N, 0, 1), \ldots, p(N, n-1, n_P)\}$, can be obtained by solving the following two equilibrium equations:

$$\begin{cases} \pi \mathbf{Q} = \vec{0}, \\ \pi \vec{1}^{\mathbf{T}} = 1, \end{cases} \tag{7}$$

where $\vec{0}$ and $\vec{1}$ are $(n_S + n)(n_P + 1)$-dimensional row vectors with all elements being 0 and 1, respectively, while the symbol $\mathbf{T}$ denotes the transpose of a matrix.

Based on this row vector, Appendix A will further derive the expected number of $N^{(0)}_{P,L}$ buffering and transmitting PPs at the beginning of a leasing period and the expected number of $N^{(0)}_{P,B}$ ones at the beginning of a buffering period.

IV. Utility Derivation

In spectrum leasing, the SN or PN, respectively, benefits from the transmission of SPs or PPs and suffers from the QoS degradation due to the temporary buffering of SPs or PPs. Thus this section first formulates the transmission revenue and buffering cost of the SN, then calculates those of the PN, and finally derives the utilities of SN and PN in the TASL.

In this derivation, denote by e_S or e_P the revenue for the SN or PN to transmit one SP or PP, respectively, and by c_S or c_P the cost for the SN or PN to buffer one SP or PP for a unit time. In practice, e_S or e_P can be interpreted as the quality-of-service (QoS) satisfaction of secondary or primary transmission, respectively, and c_S or c_P as the delay sensitivity of secondary or primary transmission [34]. Depending on the specific type of traffic, e.g., voice, video, and data, there exist various existing approaches, e.g., [34], for the SN to estimate e_S and c_S and the PN to estimate e_P and c_P independently.

A. Buffering Cost and Transmission Revenue of SN

In a buffering period, since the SN does not transmit any SP, it should achieve zero transmission revenue. On the other hand, the SN normally incurs non-zero buffering cost in both leasing and buffering periods and also achieves non-zero transmission revenue in each leasing period.

As the expected buffering time of the n SPs accumulated by the SN in a buffering period can be expressed as

$$E[B_{S,B}] = \left(\frac{1}{\lambda_S} + \frac{2}{\lambda_S} + \cdots + \frac{n-1}{\lambda_S}\right) = \frac{n(n-1)}{2\lambda_S}, \tag{8}$$

the expected buffering cost of the SN in a buffering period should be $c_S E[B_{S,B}]$.

Meanwhile, to derive the expected buffering cost of the SN in one leasing period, we can divide this period into multiple non-overlapping sub-periods such that the k^{th} sub-period, where $k \geq 1$, starts from the generation of the k^{th} SP to that of the $(k+1)^{st}$ SP. By calculating the possible number of buffering SPs and the expected buffering time for each buffering SP in the k^{th} sub-period, the expected buffering time of the SN in this sub-period can be estimated. Thus the expected buffering time of the SN in one leasing period, i.e., $E[B_{S,L}]$, can finally be obtained as the summation of those in all sub-periods and the expected buffering cost of the SN in the same period should be $c_S E[B_{S,L}]$. Appendix B will detail the derivation of $E[B_{S,L}]$.

Finally, as the SN should finish transmitting all buffering and newly generated SPs in a leasing period, the expected transmission throughput of the SN, i.e., the expected number of transmitted SPs, in a leasing period is given by

$$E[R_{S,L}] = n + \lambda_S E[T_L], \tag{9}$$

where $E[T_L]$ has been expressed in (2). Thus the expected transmission revenue of the SN in a leasing period should be $e_S E[R_{S,L}]$.

B. Buffering Cost and Transmission Revenue of the PN

In a leasing or buffering period, the PN utilizes N_P or N licensed channels for PP transmissions, respectively, which may incur both buffering cost and transmission revenue.

Similar as the analysis for the expected buffering cost of the SN in one leasing period, one leasing (*resp.* buffering) period of the PN can be divided into multiple non-overlapping sub-periods such that the first sub-period is from the beginning of this leasing (*resp.* buffering) period to the generation of the first PP in this period, while the k^{th} sub-period, where $k \geq 2$, is the time interval between the generation of the $(k-1)^{st}$ and k^{th} PPs. By calculating the possible number of buffering PPs and the expected buffering time for each buffering PP in the k^{th} sub-period, where $k \geq 1$, the expected buffering time of the PN in this sub-period can be estimated. Thus the expected buffering time of the PN in one leasing (*resp.* buffering) period, i.e., $E[B_{P,L}]$ (*resp.* $E[B_{P,B}]$), can finally be obtained as the summation of those in all sub-periods and the expected buffering cost of the PN in the same period should be $c_P E[B_{P,L}]$ (*resp.*

$$U_{PN} = \frac{e_P\left(E[R_{P,L}] + E[R_{P,B}]\right) - c_P\left(E[B_{P,B}] + E[B_{P,L}]\right) + (K + pE[T_L])N_S}{E[T_{pd}]} \tag{13}$$

$c_P E[B_{P,B}]$). Appendix C will detail the derivation of $E[B_{P,L}]$ (*resp*. $E[B_{P,B}]$).

Finally, recall from the end of Section III-B that a leasing period starts with an expected number of $N_{P,L}^{(0)}$ buffering and transmitting PPs and ends with an expected number of $N_{P,B}^{(0)}$ ones. Thus the expected transmission throughput of PN in a leasing period should be

$$E[R_{P,L}] = \lambda_P E[T_L] + N_{P,L}^{(0)} - N_{P,B}^{(0)}, \tag{10}$$

and that in a buffering period should be

$$E[R_{P,B}] = \lambda_P E[T_B] + N_{P,B}^{(0)} - N_{P,L}^{(0)}, \tag{11}$$

where $E[T_B]$ has been expressed in (1). This implies that the expected transmission revenue of the PN in a leasing or buffering period is $e_P E[R_{P,L}]$ or $e_P E[R_{P,B}]$, respectively.

C. Average Utilities of SN and PN

To compensate the PN for spectrum leasing, the SN should pay a price to the PN for each leasing period, which can also be interpreted as the cooperative service provided by the SN. This payment consists of two parts, one being proportional to the time length of a leasing period and the other independent of it. That is, if the SN leases N_S channels from the PN in a leasing period at a price p per channel per unit time, then the payment proportional to the expected time length $E[T_L]$ of a leasing period will be $pN_S E[T_L]$. On the other hand, to compensate the signaling overhead for the PN to clear the N_S leased channels before a leasing period begins, the SN should pay a fixed charge K to the PN for clearing each leased channel, which is independent of $E[T_L]$. Thus the expected payoff of the SN in a leasing period is $(K + pE[T_L])N_S$.

Following this clue, the present paper adopts the average utility per unit time, instead of the expected utility [16]–[19], to evaluate the performance of SN and PN. More specifically, the average utility of the SN in TASL can be expressed as

$$U_{SN} = \frac{e_S E[R_{S,L}] - c_S\left(E[B_{S,B}] + E[B_{S,L}]\right) - (K + pE[T_L])N_S}{E[T_{pd}]}, \tag{12}$$

where $E[B_{S,B}]$ and $E[R_{S,L}]$ have appeared in (8) and (9), respectively, while $E[B_{S,L}]$ will be expressed as (22) in Appendix B. Meanwhile, the average utility of the PN in TASL can be expressed as (13), shown at the top of the page, where $E[R_{P,L}]$ and $E[R_{P,B}]$ have appeared in (10) and (11), respectively, while $E[B_{P,L}]$ or $E[B_{P,B}]$ will be expressed as (23) or (24) in Appendix C.

The reason for adopting the average utility instead of the expected utility here lies in the fact that the time length of both leasing and buffering periods, i.e., T_L and T_B, are variable over time and so is the time length for each round of leasing and buffering periods, i.e., $T_{pd} (= T_L + T_B)$. Thus, by dividing the expected utility of SN or PN in one round with T_{pd}, the average utility of SN or PN can eliminate the effect of variable time length on spectrum leasing and hence qualifies as a fair criterion for comparing the performance of various spectrum leasing schemes with different values of T_{pd}.

V. Game-Theoretical Formulation of TASL

From Section IV, when various PN and SN parameters, i.e., $N, K, \lambda_S, \lambda_P, \mu_S, \mu_P, e_S, e_P, c_S, c_P$, and n_P, are given, the average utilities of PN and SN in TASL will be determined by the number N_S of the leased channels, the leasing price p, and the maximum number of n buffering SPs that the SN can accumulate in a buffering period. As the PN and SN normally belong to different service providers and hence have different interests, they should negotiate with each other for the selection of N_S, p, and n so as to maximize their respective long-term benefits subject to the QoS requirements on primary and secondary transmissions, e.g., (3) and (4). To facilitate this negotiation process, this section develops a non-cooperative game model for the determination of N_S, p, and n.

In this game, the PN and SN act as the *leader* and *follower* of spectrum leasing, respectively, such that the PN should always determine the parameters N_S and p before the SN can select the parameter n. This game model can be characterized by a triplet $\{TN, (S_i)_{i\in TN}, (U_i)_{i\in TN}\}$, where

- $TN = \{PN, SN\}$ is the set of two players, i.e., the PN and SN, in this game;
- For each $i \in TN$, S_i is the set of candidate strategies for the player i. That is, $S_{SN} = \{0, 1, \ldots, n_S\}$ and $S_{PN} = \{(N_S, p) | \lceil \frac{\lambda_S E[T_{pd}]}{\mu_S E[T_L]} \rceil \leq N_S \leq \lfloor \frac{NE[T_{pd}]}{E[T_L]} - \frac{\lambda_P E[T_{pd}]}{\mu_P E[T_L]} \rfloor, p \in \{0, \triangle p, 2\triangle p, \ldots\}\}$, where $\triangle p$ is the minimal pricing interval and the lower and upper bounds of N_S have been given by (3) and (4), respectively;
- For each $i \in TN$, U_i is the average utility of the player i, i.e., U_{SN} in (12) and U_{PN} in (13).

A. The SN Strategy

Given a certain strategy $(N_S, p) \in S_{PN}$, the set of the optimal SN strategies is defined as

$$\begin{aligned} S_{SN}^*(N_S, p) = \{&n^* \in S_{SN} | U_{SN}^*(N_S, p, n^*) \\ &\geq (U_{SN}(N_S, p, n))^+, \forall n \in S_{SN}\}, \end{aligned} \tag{14}$$

where $(x)^+ = x$ for $x \geq 0$ or $(x)^+ = 0$ for $x < 0$, $U_{SN}^*(N_S, p, n^*)$ is the maximal average utility of SN given that the PN selects (N_S, p), and the constraint $U_{SN}^*(N_S, p, n^*) \geq 0$ guarantees that the SN always have an incentive to participate in the spectrum leasing. Note that the SN may have multiple choices of $n^* \in S_{SN}^*(N_S, p)$, i.e., $|S_{SN}^*(N_S, p)| > 1$, to maximize its average utility. In general, the smaller the maximal number of buffering SPs that the SN accumulates in a buffering period, the less the average time delay for SP transmission, and the better the QoS of secondary transmission. Thus the SN is willing to apply the following rule for selecting a unique n from multiple choices:

Algorithm 1: Derivation of the equilibrium solution for the proposed TASL.

1: Initialize $p = \triangle p$, $N_S = \lceil \frac{\lambda_S E[T_{pd}]}{\mu_S E[T_L]} \rceil$, $\hat{U}_{PN} = 0$, and $\hat{S}_{PN} = \emptyset$.
2: Generate the set $S^*_{SN}(N_S, p)$ of optimal SN strategies according to (12) and (14). If $S^*_{SN}(N_S, p) = \emptyset$, then go to Step 5; else, set $n^*_{\min} = \min_{n^* \in S^*_{SN}(N_S,p)} n^*$.
3: Calculate the average utility of PN, i.e., $U^*_{PN}(N_S, p, n^*_{\min})$, by (13). If $U^*_{PN} > \hat{U}_{PN}$, then set $\hat{U}_{PN} = U^*_{PN}$ and $\hat{S}_{PN} = \{(N_S, p)\}$; else, if $U^*_{PN} = \hat{U}_{PN}$, then set $\hat{S}_{PN} = \hat{S}_{PN} \cup \{(N_S, p)\}$.
4: Set $p = p + \triangle p$ and return to Step 2.
5: If $N_S < \lfloor \frac{N E[T_{pd}]}{E[T_L]} - \frac{\lambda_P E[T_{pd}]}{\mu_P E[T_L]} \rfloor$, then set $N_S = N_S + 1$ as well as $p = \triangle p$ and return to Step 2.
6: If $\hat{S}_{PN} \neq \emptyset$, then output $S^* = \{(N^*_{S,\min}, p^*, n^*_{\min}) | N^*_{S,\min} = \min_{(N_S,p) \in \hat{S}_{PN}} N_S\}$ as the equilibrium solution of the TASL; else, output $S^* = \emptyset$.

A) Among all choices of $n^* \in S^*_{SN}(N_S, p)$, the SN should always prefer the smallest one, i.e., $n^*_{\min} = \min_{n^* \in S^*_{SN}(N_S,p)} n^*$.

B. The PN Strategy

In view of the best response $n^*_{\min}$ of the SN, the set of the optimal PN strategies is defined as

$$S^*_{PN}(n^*_{\min}) = \big\{(N^*_S, p^*) \in S_{PN} | U^*_{PN}(N^*_S, p^*, n^*_{\min}) \geq (U_{PN}(N_S, p, n^*_{\min}))^+, \forall (N_S, p) \in S_{PN}\big\}, \tag{15}$$

where $U^*_{PN}(N^*_S, p^*, n^*_{\min})$ is the maximal average utility of PN given that the SN selects $n^*_{\min}$, while the constraint $U^*_{PN}(N^*_S, p^*, n^*_{\min}) \geq 0$ serves as an incentive for the PN to participate the spectrum leasing. Note that the PN may have multiple choices of $N^*_S \in S^*_{PN}$, i.e., $|S^*_{PN}| > 1$, to maximize its average utility. In this case, because the PN normally prefers less leased channels to more leased channels for achieving a better QoS guarantee for primary transmission, it is willing to apply the following rule for determining a unique N_S in spectrum leasing:

B) Among all choices of $(N^*_S, p^*) \in S^*_{PN}(n^*_{\min})$, the PN prefers the one with the smallest value of N^*_S, i.e., $N^*_{S,\min} = \min_{(N^*_S,p^*) \in S^*_{PN}(n^*_{\min})} N^*_S$.

C. Equilibrium Solution of Spectrum Leasing

The optimal strategies $n^*_{\min}$ in (14) and $(N^*_{S,\min}, p^*)$ in (15) together constitute a Stackelberg equilibrium. To determine this equilibrium, a general method is the backward induction [35], which first calculates the best response n of the SN in terms of the parameters N_S and p and then backtracks to the determination of the optimal (N_S, p) of the PN. As it is difficult to express n in terms of (N_S, p) by (12), we further develop Algorithm 1 for the PN to derive the equilibrium of the proposed Stackelberg game.

In this algorithm, the PN should search every possible strategy of (N_S, p) for generating the corresponding best response $n^*_{\min}$ of the SN in Step 2, based on which the PN can recursively update its maximal average utility in Step 3. As this updating process always starts with $N_S = \lceil \frac{\lambda_S E[T_{pd}]}{\mu_S E[T_L]} \rceil$ and ends with $N_S = \lfloor \frac{N E[T_{pd}]}{E[T_L]} - \frac{\lambda_P E[T_{pd}]}{\mu_P E[T_L]} \rfloor$, the optimal strategies of PN and SN derived by Algorithm 1, if available, can guarantee the basic QoS requirements (3) and (4).

TABLE II
PARAMETERS FOR ANALYTICAL COMPUTATION AND NUMERICAL SIMULATION

Parameters	Values
λ_S or λ_P	0.6 packets/s or 2 packets/s
μ_S or μ_P	0.7 packets/s or 1.1 packets/s
n_S or n_P	1000 or 1000
c_S or c_P	0.1 or 0.1
e_S or e_P	4 or 1
K	0.1

Theorem 1: Whenever Algorithm 1 generates a non-empty solution set S^*, it will always yield a unique equilibrium solution for the proposed TASL.

Proof: By (14) and (15), whenever the solution set S^* is not empty, both S^*_{SN} and S^*_{PN} will also be non-empty. By the rule (B), given an optimal SN strategy $n^*_{\min}$, the PN will always set its optimal N_S as $N^*_{S,\min} = \min_{(N^*_S,p^*) \in S^*_{PN}(n^*_{\min})} N^*_S$ and hence there may exist multiple 2-tuples, say, $(N^*_{S,\min}, \triangle p), (N^*_{S,\min}, 2\triangle p), \ldots, (N^*_{S,\min}, k\triangle p)$, which can guarantee $U^*_{PN} > 0$. In this case, because the average utility of PN is a linearly increasing function of p by (13), the PN will always choose the highest price $p^*_{\max} = k\triangle p$ to maximize its average utility. On the other hand, given the optimal PN strategy $(N^*_{S,\min}, p^*_{\max})$, the SN under the rule (A) should always choose the smallest one among all $n^* \in S^*_{SN}(N^*_{S,\min}, p^*_{\max})$ to maximize its average utility. Thus (A) and (B) together guarantee a unique equilibrium solution $(N^*_{S,\min}, p^*_{\max}, n^*_{\min})$ for the TASL. ■

VI. NUMERICAL SIMULATION

This section first validates the theoretical analysis for the proposed TASL, i.e., the Markov chain in Section III and the average utilities of PN and SN in Section IV, and then compares the performance of TASL and the existing spectrum leasing schemes via numerical simulation.

A. Validation of Theoretical Analysis

To validate the theoretical analysis of the proposed TASL, we set the total number of licensed channels as $N = 3$, the number of leased channels as $N_S = 1$, and other leasing parameters as summarized in Table II. Under this parameter setting, both constraints (3) and (4) can be satisfied. Each simulation is carried out for at least 1000 alternations of leasing and buffering periods.

Fig. 3(a) depicts the expected time length of one buffering or leasing period, i.e., $E[T_B]$ in (1) or $E[T_L]$ in (2), in terms of the maximal number of buffering SPs that the SN accumulates in one buffering period, i.e., n. It shows that, when n becomes large, both $E[T_L]$ and $E[T_B]$ will increase accordingly. This can be explained by the fact that the more SPs that the SN buffers at the end of a buffering period, the longer the time length for

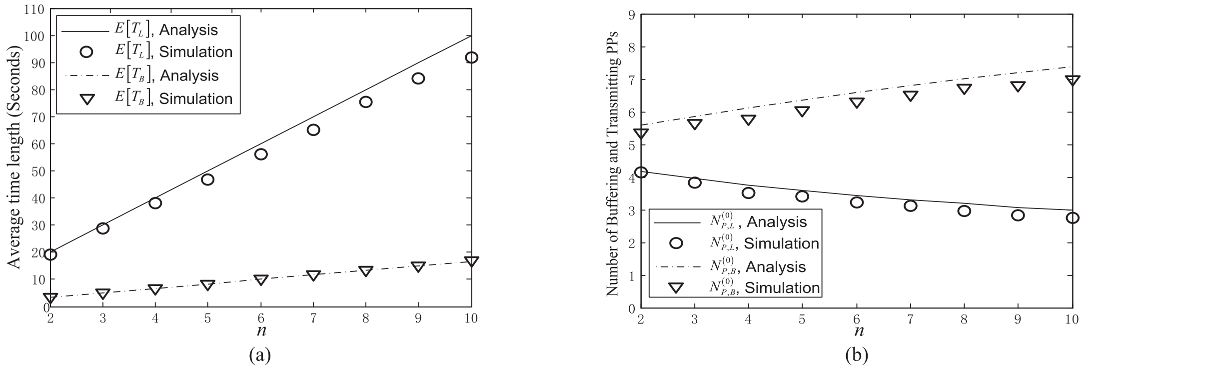


Fig. 3. (a) The expected time length of one leasing or buffering period, i.e., $E[T_L]$ or $E[T_B]$, versus n. (b) The expected number of buffering and transmitting PPs at the beginning of one leasing or buffering period, i.e., $N_{P,L}^{(0)}$ or $N_{P,B}^{(0)}$, versus n.

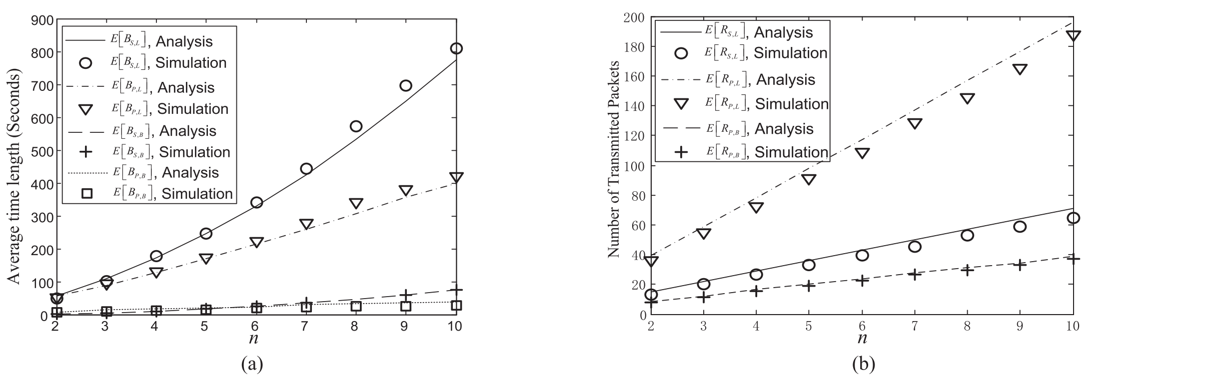


Fig. 4. (a) The expected buffering time for all SPs or PPs in one leasing or buffering period versus n. (b) The expected number of SPs or PPs transmitted in one leasing or buffering period versus n.

the SN to accumulate them in this buffering period and finish transmitting them in the ensuing leasing period.

Meanwhile, Fig. 3(b) illustrates the expected number of PPs at the beginning of one leasing or buffering period, i.e., $N_{P,L}^{(0)}$ in (17) or $N_{P,B}^{(0)}$ in (18), in terms of n. It shows that, when n becomes large, $N_{P,B}^{(0)}$ will increases and $N_{P,L}^{(0)}$ will decrease. This can be explained by the fact that, as the PN utilizes less channels for PP transmission in a leasing period than in a buffering period, it normally buffers more PPs at the beginning of the ensuing buffering period than at the beginning of the ensuing leasing period. Thus $N_{P,B}^{(0)}$ should always be larger than $N_{P,L}^{(0)}$. Moreover, as both $E[T_B]$ and $E[T_L]$ increase with n, so does the gap between $N_{P,B}^{(0)}$ and $N_{P,L}^{(0)}$.

Fig. 4(a) depicts the expected buffering time for all SPs or PPs in one leasing or buffering period, i.e., $E[B_{S,B}]$ in (8), $E[B_{S,L}]$ in (22), $E[B_{P,L}]$ in (23), and $E[B_{P,B}]$ in (24), in terms of the parameter n. It shows that, when n becomes large, $E[B_{S,B}], E[B_{S,L}], E[B_{P,L}]$, and $E[B_{P,B}]$ will increase accordingly. Meanwhile, Fig. 4(b) depicts the expected number of SPs and PPs transmitted in one leasing or buffering period, i.e., $E[R_{S,L}]$ in (9), $E[R_{P,L}]$ in (10), and $E[R_{P,B}]$ in (11), in terms of n. It shows that the expected number of SPs or PPs transmitted in a leasing or buffering period will also increase with n. Moreover, Fig. 4(a) and (b) together show that, as n increases, the expected buffering time of all SPs increases faster than the expected number of transmitted SPs. Thus the TASL parameters should be carefully selected for guaranteeing both PN and SN to achieve positive average utilities and hence have incentive to join spectrum leasing.

Finally, both Figs. 3 and 4 show that the theoretical results for various metrics of TASL match with their numerical simulation values well. This verifies the effectiveness of the Markov chain formulated in Section III and the average utilities derived in Section IV.

B. Comparison Between the TASL and Existing Spectrum Leasing Scheme

As a comparison for the proposed TASL, we consider a so-called *partially traffic-adaptive spectrum leasing* (PTASL) scheme, which generalizes a class of the existing spectrum leasing schemes, e.g., [14]–[19], for cooperative relay and allows one PN and one SN to negotiate a fixed time length for all leasing periods according to the statistical information of secondary traffics.

In the PTASL, when the SN leases $N_S \in [1, N]$ licensed channels from the PN, the infinite time of these channels will be synchronously divided into multiple frames with a uniform time length T_{pd} and each frame will be composed of a buffering period for the SN to buffer SPs and a leasing period for the SN to transmit SPs over the N_S leased channels. However, different from the TASL in which the time length of each buffering or leasing period is variable according to the dynamic generation of SPs, the PTASL always sets one fixed time length $T_L = \alpha T_{pd}$ for all leasing periods, where $\alpha \in [0, 1]$, and the other fixed time length $T_B = (1 - \alpha)T_{pd}$ for all buffering periods before the

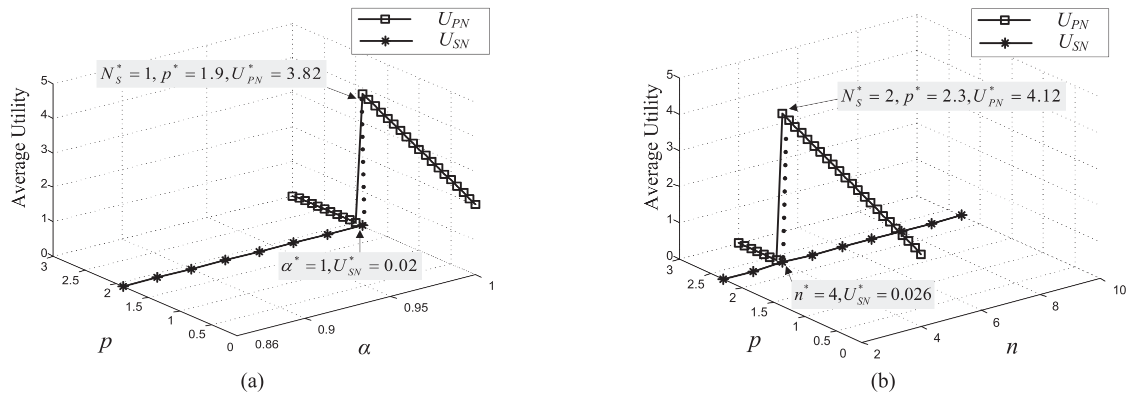


Fig. 5. (a) The average utilities of SN and PN, i.e., U_{SN} and U_{PN}, versus α and p in the PTASL. (b) The average utilities U_{SN} and U_{PN} versus n and p in the TASL.

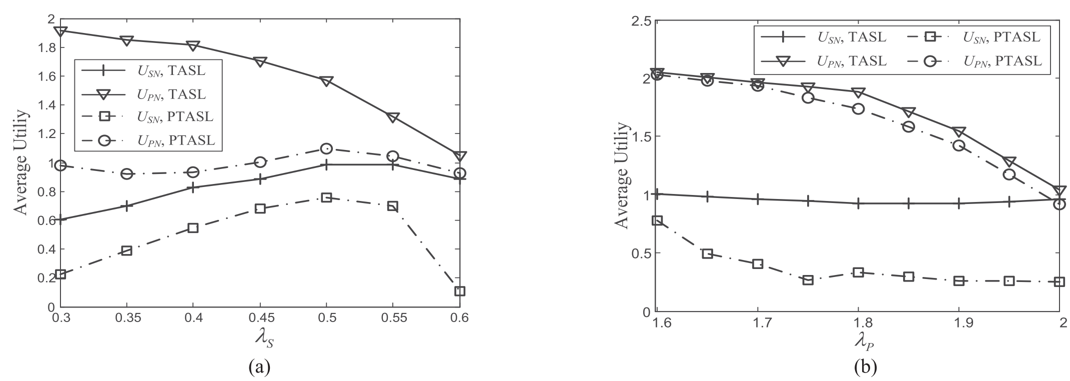


Fig. 6. The average utilities of SN and PN, i.e., U_{SN} and U_{PN}, are depicted in terms of (a) the average generation rate of SPs, i.e., λ_S, and (b) the average generation rate of PPs, i.e., λ_P, where $p = 1$.

spectrum leasing begins. This transforms the QoS constraints (3) and (4) into

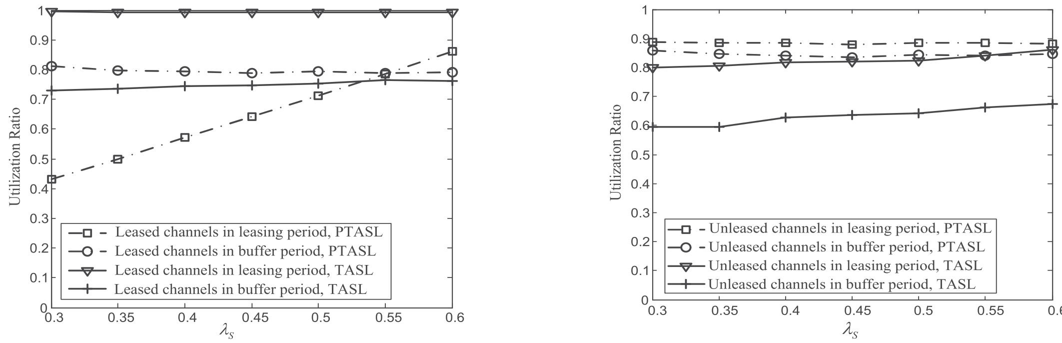
$$\left\lceil \frac{\lambda_S}{\mu_S \alpha} \right\rceil \le N_S \le \left\lfloor \frac{N}{\alpha} - \frac{\lambda_P}{\mu_P \alpha} \right\rfloor. \tag{16}$$

Similar as TASL, we can also formulate the PTASL as a non-cooperative Stackelberg game, where the PN chooses N_S and p before the determination of α by the SN. Thus, by replacing n with α, Algorithm 1 can also derive the equilibrium solution N_S, p and α for PTASL.

The comparison of the equilibrium solutions of TASL and PTASL follows the parameter setting in Table II, adopts $T_{pd} =$ 15 seconds for PTASL, and sets the total number of licensed channels as $N = 4$. According to (16) and the constraint $\alpha \in [0, 1]$, the feasible range of α in the PTASL is $[0.86, 1]$ for $N_S =$ 1, $[0.43, 1]$ for $N_S = 2$, $[0.29, 0.73]$ for $N_S = 3$, or $[0.22, 0.55]$ for $N_S = 4$.

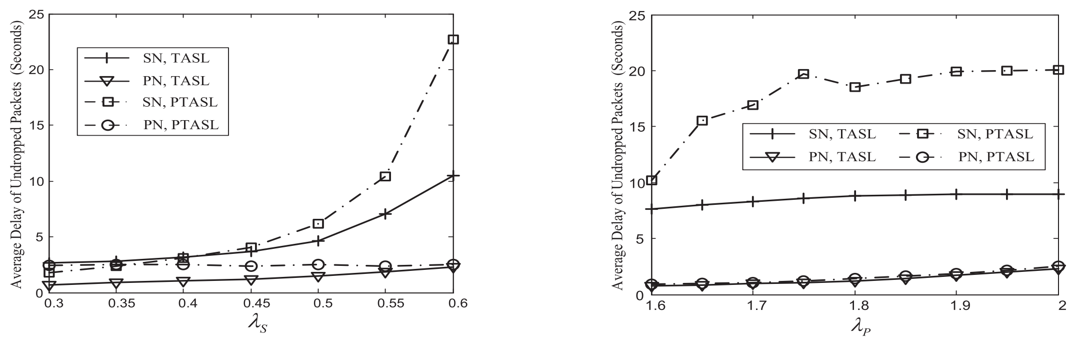
Under this parameter setting, Fig. 5(a) depicts the average utilities of SN and PN, i.e., U_{SN} and U_{PN}, in the PTASL in terms of α and p, while Fig. 5(b) illustrates U_{SN} and U_{PN} in the TASL in terms of n and p. It shows that the unique equilibrium in the PTASL solution is $N_S^* = 1, p^* = 1.9$, and $\alpha = 1$, at which both SN and PN can maximize their average utilities by leasing one licensed channel all the time for SP transmission without inserting any buffering period. The average utilities of PN and SN at this equilibrium of PTASL are 3.82 and 0.02, respectively. On the other hand, the unique equilibrium solution in the TASL is $N_S^* = 2, p^* = 2.3$, and $n^* = 4$, which yields the average PN and SN utilities of 4.12 and 0.026, respectively. As the TASL can afford PN and SN with larger equilibrium average utilities than the PTASL, they are better off to adopt the TASL instead of the PTASL. That is, compared with leasing one licensed channel for SP transmission all the time, both PN and SN can benefit more by inserting one buffering period between any two adjacent leasing periods and adapt both leasing and buffering periods to the real-time generation and transmission of SPs.

Fig. 6(a) depicts the average utilities of PN and SN in terms of the average generation rate of SPs, i.e., λ_S, under both TASL and PTASL and Fig. 6(b) depicts the same utilities in terms of the average generation rate of PPs, i.e., λ_P. They show that, when λ_S becomes large, the average utility of SN, i.e., U_{SN}, will fluctuate. The reason lies in the following three aspects. First, the increasing of λ_S will enlarge the expected time length of one leasing period, i.e., $E[T_L]$, by (2), increase the expected number of SPs transmitted in one leasing period, i.e., $E[R_{S,L}]$, by (9), and reduce the expected buffering time of the SN in one leasing period, i.e., $E[B_{S,B}]$, by (8). Second, given a fixed transmission rate of SPs, e.g., μ_S, because the increasing of λ_S will make each newly generated SP in one leasing period to see more buffering SPs, the expected buffering time of the SN, i.e., $E[B_{S,L}]$, in this period will also increase. Third, when $E[T_L]$ becomes large, the payment $(K + pE[T_L])N_S$ from the SN to the PN will also increase. By (12), these effects of increasing λ_S together will make U_{SN} fluctuant. Regardless of the value of λ_S or λ_P, Fig. 6(a) and (b) show that the TASL can always achieve



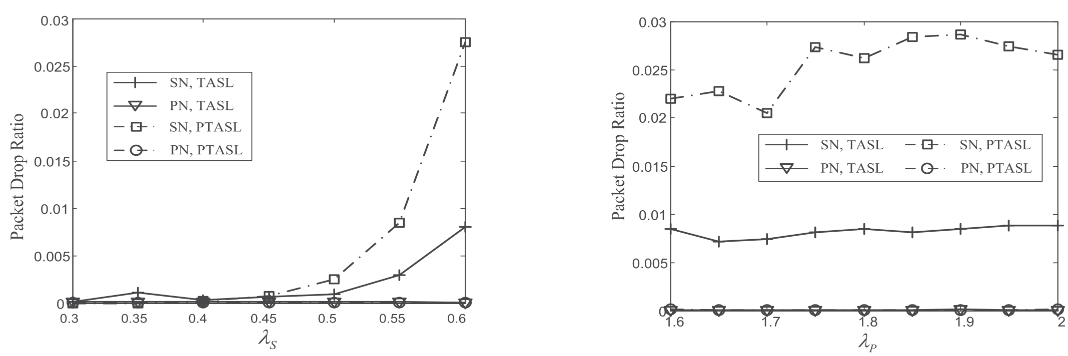
(a) (b)

Fig. 7. (a) The utilization ratios of leased channels in leasing and buffering periods versus λ_S. (b) The utilization ratios of unleased channels in leasing and buffering periods versus λ_S.



(a) (b)

Fig. 8. The average delays of undropped SPs and PPs are depicted in terms of (a) λ_S and (b) λ_P, where $p = 1$.



(a) (b)

Fig. 9. The packet drop ratios of SN and PN are depicted in terms of (a) λ_S and (b) λ_P, where $p = 1$.

larger average utilities for both PN and SN than the PTASL. Moreover, because the PN always makes its decision on p and N_S before the SN can choose n in the TASL or α in the PTASL, it can enjoy the so-called *first mover* advantage [35] by always achieving a larger average utility than the SN.

Fig. 7 illustrates the utilization ratio of leased and unleased channels in terms of λ_S at the equilibrium solutions of PTASL and TASL. Among them, Fig. 7(a) shows that, regardless of the value of λ_S, the TASL can always achieve much larger utilization ratio of N_S leased channels in leasing periods than the PTASL. Thus the TASL enables the SN to transmit SPs at relatively higher spectrum efficiency and better transmission performance than the PTASL. On the other hand, Fig. 7(b) shows that, compared with the PTASL, the TASL can save the PN with more time-frequency resource of unleased channels for guaranteeing the QoS requirement of PP transmission and hence make the PN more willing to join spectrum leasing.

Finally, to further compare the transmission performance of PN and SN under the TASL and PTASL in those applications that allow limited packet buffering time, we impose an additional constraint for the PN or SN, respectively, to drop the PP or SP that has being buffered for more than 30 seconds. Fig. 8(a) or (b) depicts the average delays of undropped PPs and SPs in terms of the parameter λ_S or λ_P, respectively, while Fig. 9(a) or (b) illustrates the packet drop ratios of PN and SN in terms of λ_S or λ_P. These figures show that the PN experiences shorter transmission delay and lower packet drop ratio than the SN in both TASL and PTASL. This can again be explained by the first mover advantage of PN in the Stackelberg gaming. Moreover, Figs. 8 and Fig. 9 together show that the SN in the TASL can achieve shorter average delay of undropped SPs and lower packet drop ratio than the SN in the PTASL for all $\lambda_P \in [1.6, 2]$ and especially when λ_S becomes large. Hence, compared with the PTASL, the TASL is more suitable for newly emerging applications, e.g., mobile games and e-health, which are sensitive to the network performance of transmission delay and the overhead of packet buffering.

VII. Conclusion

This paper proposes a novel traffic-adaptive spectrum leasing (TASL) scheme by allowing one secondary network (SN) to lease a number of licensed channels from one primary network (PN) for transmitting the dynamically generated secondary packets (SPs) in a fully adaptive way. By establishing a three-dimensional Markov chain model, we derive the average utilities of PN and SN in the TASL, develop a non-cooperative Stackelberg game model for the PN to determine the leasing price and the number of leased channels before the SN can choose the maximal number of buffering SPs in each buffering period, and propose two specific rules for PN and SN to achieve a unique equilibrium solution for this game. Numerical simulation validates this theoretical analysis and shows that, compared with the existing partially traffic-adaptive spectrum leasing (PTASL) scheme that presets a fixed leasing time length, the proposed TASL can effectively improve the utilization ratio of leased channels as well as afford both PN and SN with larger average utilities, smaller packet drop ratios, and shorter average delays of undropped packets. Therefore, the proposed TASL is more suitable for newly emerging applications, which are sensitive to the transmission delay and buffering overhead of data traffic, than the existing PTASL. One possible future extension of TASL then is to more complicated scenarios of traffic-adaptive spectrum leasing between one PN and multiple SNs as well as that between multiple PNs and multiple SNs.

Appendix A
Expected Number of PPs at the Beginning of a Buffering or Leasing Period

In the TASL, while the number of transmitting and buffering SPs at the beginning of a leasing or buffering period is fixed as n or 0, respectively, that of transmitting and buffering PPs at the same time point is variable. Nevertheless, its expected value in the long run can be derived as follows.

Denote by i the number of transmitting and buffering PPs in the PN when the SN generates the $(n-1)^{\text{st}}$ SP in a buffering period. As the expected time length for the SN to generate a new SP is $1/\lambda_S$, the expected number of transmitting and buffering PPs in the PN at the beginning of the ensuing leasing period will be $n_L[i] = i - (i\frac{\mu_P}{\lambda_S} - \frac{\lambda_P}{\lambda_S})^+$ for $0 \le i \le N-1$ or $n_L[i] = i - (N\frac{\mu_P}{\lambda_S} - \frac{\lambda_P}{\lambda_S})^+$ for $i \ge N$, where $\frac{\mu_P}{\lambda_S}$ denotes the expected number of PPs successfully transmitted at a licensed channel during the time interval between the generation of the $(n-1)^{\text{st}}$ SP in a buffering period and that of the first SP in the next leasing period, $\frac{\lambda_P}{\lambda_S}$ the expected number of new PPs generated in the same interval, and $(x)^+ = x$ for $x \ge 0$ or $(x)^+ = 0$ for $x < 0$. Therefore, the expected number of PPs at the beginning of a leasing period can be expressed as:

$$N_{P,L}^{(0)} = \frac{\sum_{i=0}^{n_P} n_L[i]p(N, n-1, i)}{\sum_{i=0}^{n_P} p(N, n-1, i)}. \tag{17}$$

Similarly, the expected number of transmitting and buffering PPs at the beginning of a buffering period can be expressed as:

$$N_{P,B}^{(0)} = \frac{\sum_{j=0}^{n_P} n_B[j]p(N_P, 1, j)}{\sum_{j=0}^{n_P} p(N_P, 1, j)}, \tag{18}$$

where $n_B[j]$ denotes the number of transmitting and buffering PPs at the beginning of a buffering period given that there exist $j \in [0, n_P]$ transmitting and buffering PPs at the first time when the SN contains only one transmitting SP in the last leasing period. Similar as $n_L[i]$, we have $n_B[j] = j - E[W_1](j\mu_P - \lambda_P)^+$ for $0 \le j \le N_P - 1$ or $n_B[j] = j - E[W_1](N_P\mu_P - \lambda_P)^+$ for $j \ge N_P$, where $E[W_1]$ matches with $E[T_L]$ in (2) with $n = 1$.

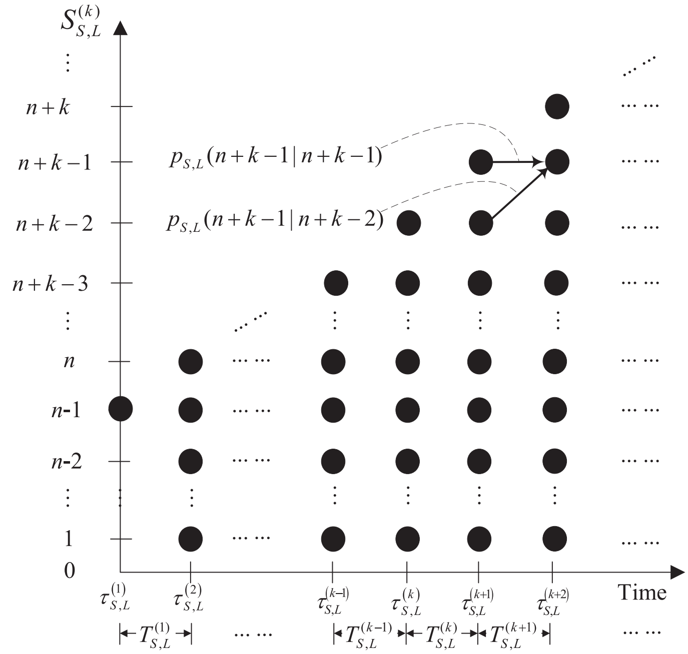


Fig. 10. Possible number of buffering and transmitting SPs seen by one SP newly generated in a leasing period, where each black circle denotes one possible number.

Appendix B
Expected Buffering Time of SN in a Leasing Period

Denote by $n_{S,L}$ the expected number of SPs newly generated by the SN in a leasing period and by $\tau_{S,L}^{(k)}$, where $k = 1, 2, \ldots$, the generation time of the k^{th} SP in a leasing period. Thus $n_{S,L} = \lceil \lambda_S E[T_L] \rceil$. Let $S_{S,L}^{(k)}$ be the total number of transmitting and buffering SPs seen by the k^{th} newly generated SP in a leasing period. From Fig. 10, the beginning time of a leasing period should be $\tau_{S,L}^{(1)}$ and the SP newly generated at $\tau_{S,L}^{(1)}$ should see exactly $n-1$ buffering SPs and zero transmitting SP. Following this clue, Fig. 10 demonstrates the possible number of SPs seen by each SP newly generated in a leasing

$$p(x|y) = \begin{cases} 0, & \text{if } x > y+1. \\ \displaystyle\int_0^\infty \binom{y+1}{x} (y - e^{-\mu t})^{y+1-x} (e^{-\mu t})^x \lambda e^{-\lambda t} dt, & \text{if } M \geq y+1 \geq x. \\ \displaystyle\int_0^\infty e^{-M\mu t} \frac{(M\mu t)^{y+1-x}}{(y+1-x)!} \lambda e^{-\lambda t} dt, & \text{if } y+1 \geq x \geq M. \\ \displaystyle\int_0^\infty \int_0^t \binom{M}{x} (1 - e^{-\mu(t-\tau)})^{M-x} (e^{-\mu(t-\tau)})^x M\mu e^{-M\mu\tau} \frac{(M\mu\tau)^{y-M}}{(y-M)!} \lambda e^{-\lambda t} d\tau dt, & \text{if } x < M \leq y+1. \\ 0, & \text{if } x > y+1. \end{cases} \quad (20)$$

period. Thus a leasing period of the SN can be divided into multiple disjoint sub-periods $T_{S,L}^{(1)}, T_{S,L}^{(2)}, \ldots, T_{S,L}^{(n_{S,L})}$, where $T_{S,L}^{(k)} = [\tau_{S,L}^{(k)}, \tau_{S,L}^{(k+1)})$ and $E[T_{S,L}^{(k)}] = \frac{1}{\lambda_S}$ for each $k \geq 1$. Since only one SP is generated during each $T_{S,L}^{(k)}$, we have $S_{S,L}^{(1)} = n-1, S_{S,L}^{(k+1)} \leq S_{S,L}^{(k)} + 1$, and $S_{S,L}^{(k)} \in \{1, 2, \ldots, k+n-2\}$, where $S_{S,L}^{(k)} = k+n-2$ means that the SN does not finish any SP transmission before the generation of the k^{th} new SP in a leasing period.

From Fig. 10, the stationary probability of $S_{S,L}^{(k)}$ for $k \geq 2$ can be obtained by recursively applying the following formula

$$p\left(S_{S,L}^{(k)} = x\right) = \sum_{y=1}^{n+k-3} p\left(S_{S,L}^{(k-1)} = y\right) \times p_{S,L}\left(S_{S,L}^{(k)} = x | S_{S,L}^{(k-1)} = y\right), \quad (19)$$

where $p_{S,L}(x|y)$ denotes the conditional probability that, when a SP newly generated in a leasing period sees exactly $y(\geq 1)$ buffering and transmitting SPs, the next newly generated SP will see exactly $x(\geq 1)$ buffering and transmitting SPs. The initial conditions for this recursive formula are $p(S_{S,L}^{(1)} = n-1) = 1$. Because $p(S_{S,L}^{(k)} = 0) = 0$ and $p(S_{S,L}^{(k)} > S_{S,L}^{(k-1)} + 1) = 0$ for any $k \geq 2$, we have $p_{S,L}(S_{S,L}^{(k)} = x | S_{S,L}^{(k-1)} = y) = \frac{p(x|y)}{1-p(0|y)}$, where $p(x|y)$ has been given by [33] and is expressed as the formula (20) shown at the top of the page, with $\lambda = \lambda_S, \mu = \mu_S$ and $M = N_S$.

The expected buffering time $E[B_{S,L}]$ of the SN in a leasing period then can be derived by summing the expected buffering time in the sub-periods $T_{S,L}^{(1)}, T_{S,L}^{(2)}, \ldots$ During the sub-period $T_{S,L}^{(k)}$ for $k \geq 1$, if $S_{S,L}^{(k)} < N_S$, then the constraint $S_{S,L}^{(k+1)} \leq S_{S,L}^{(k)} + 1$ will imply that the N_S leased channels are enough for SP transmission and hence the SN does not need to buffer any SP during this sub-period; else, the SN should always buffer at least one SP for non-zero time length in this sub-period. Thus we only need to consider the case of $S_{S,L}^{(k)} \geq N_S$ and derive $E[B_{S,L}]$ depending on whether $S_{S,L}^{(k+1)} > N_S$.

Case 1: $S_{S,L}^{(k+1)} > N_S$.

In this case, the SN during the sub-period $T_{S,L}^{(k)}$ should always buffer at least $(S_{S,L}^{(k+1)} - N_S)$ SPs and finish $(S_{S,L}^{(k)} + 1 - S_{S,L}^{(k+1)})$ SP transmissions, each of which will initiate a new transmission of one buffering SP in the same sub-period. Since the transmission of SPs at different leased channels are fully independent and the time length for transmitting one SP at each licensed channel follows a common exponential distribution, the ending time points of all $(S_{S,L}^{(k)} + 1 - S_{S,L}^{(k+1)})$ SP transmissions should be uniformly distributed within the sub-period $T_{S,L}^{(k)}$ and so are the starting time points of all $(S_{S,L}^{(k)} + 1 - S_{S,L}^{(k+1)})$ new SP transmissions in the same period. That is, the expected starting time point for the first new SP transmission during the sub-period $T_{S,L}^{(k)}$ is $\tau_{S,L}^{(k)} + \frac{T_{S,L}^{(k)}}{S_{S,L}^{(k)}+1-S_{S,L}^{(k+1)}}$, that for the second new SP transmission is $\tau_{S,L}^{(k)} + \frac{2T_{S,L}^{(k)}}{S_{S,L}^{(k)}+1-S_{S,L}^{(k+1)}}, \ldots$, and that for the $\left(S_{S,L}^{(k)} + 1 - S_{S,L}^{(k+1)}\right)^{\text{th}}$ new SP transmission is $\tau_{S,L}^{(k)} + T_{S,L}^{(k)}$. Therefore, when $S_{S,L}^{(k+1)} > N_S$, the expected buffering time during the sub-period $T_{S,L}^{(k)}$ can be estimated as

$$E\left[B_{S,L,1}^{(k)}\right] = \sum_{j=1}^{S_{S,L}^{(k)}+1-S_{S,L}^{(k+1)}} \frac{jE\left[T_{S,L}^{(k)}\right]}{S_{S,L}^{(k)} + 1 - S_{S,L}^{(k+1)}} + E\left[T_{S,L}^{(k)}\right]\left(S_{S,L}^{(k+1)} - N_S\right), \quad (21)$$

where the first item denotes the expected buffering time for the $(S_{S,L}^{(k)} + 1 - S_{S,L}^{(k+1)})$ SPs of which the transmissions begin in the sub-period $T_{S,L}^{(k)}$, while the second item is the expected total buffering time for the $(S_{S,L}^{(k+1)} - N_S)$ SPs that are always buffered by the SN during the same sub-period. By substituting $E[T_{S,L}^{(k)}] = \frac{1}{\lambda_S}$ into (21), we have $E[B_{S,L,1}^{(k)}] = \frac{1}{\lambda_S}[\frac{1}{2}(S_{S,L}^{(k)} + S_{S,L}^{(k+1)}) + 1 - N_S]$.

Case 2: $S_{S,L}^{(k+1)} \leq N_S$.

In this case, the SN should first buffer at least one SP before a certain time point $\tau_{N_S}^{(k)} \in [\tau_{S,L}^{(k)}, \tau_{S,L}^{(k+1)})$ and then no SP since then. Because of the uniform distribution of the ending time points of all $(S_{S,L}^{(k)} + 1 - S_{S,L}^{(k+1)})$ SP transmissions during the sub-period $T_{S,L}^{(k)}$, we have $\frac{\tau_{N_S}^{(k)} - \tau_{S,L}^{(k)}}{T_{S,L}^{(k)}} = \frac{S_{S,L}^{(k)}+1-N_S}{S_{S,L}^{(k)}+1-S_{S,L}^{(k+1)}}$ or, equivalently, $\tau_{N_S}^{(k)} = \frac{(S_{S,L}^{(k)}+1-N_S)T_{S,L}^{(k)}}{S_{S,L}^{(k)}+1-S_{S,L}^{(k+1)}} + \tau_{S,L}^{(k)}$. By the same derivation in Case 1, when $S_{S,L}^{(k+1)} \le N_S$, the expected buffering time incurred for the sub-period $T_{S,L}^{(k)}$ can be expressed as $E[B_{S,L,2}^{(k)}] = \sum_{j=1}^{S_{S,L}^{(k)}+1-N_S} \frac{(2j-1)E[\tau_{N_S}^{(k)} - \tau_{S,L}^{(k)}]}{2(S_{S,L}^{(k)}+1-N_S)}$.

By summing up the expected buffering time of the SN in the sub-periods $T_{S,L}^{(1)}, T_{S,L}^{(2)}, \ldots, T_{S,L}^{(n_{S,L})}$, we can further express the expected buffering time of SN during a leasing period as

$$E[B_{S,L}] = \sum_{k=1}^{n_{S,L}} \sum_{S_{S,L}^{(k)}=N_S}^{n+k-2} p\left(S_{S,L}^{(k)}\right) \left(\sum_{S_{S,L}^{(k+1)}=N_S+1}^{S_{S,L}^{(k)}+1} E\left[B_{S,L,1}^{(k)}\right] \times p\left(S_{S,L}^{(k+1)}\right) + \sum_{S_{S,L}^{(k+1)}=1}^{N_S} E\left[B_{S,L,2}^{(k)}\right] p\left(S_{S,L}^{(k+1)}\right) \right), \quad (22)$$

where the first or second item in the bracket denotes the expected buffering time of the SN during the sub-period $T_{S,L}^{(k)}$ when $S_{S,L}^{(k+1)} > N_S$ or $S_{S,L}^{(k+1)} \le N_S$, respectively.

Appendix C
Expected Buffering Time of PN in Leasing and Buffering Periods

Denote by $\tau_{P,L}^{(k)}$ (*resp.* $\tau_{P,B}^{(k)}$) the time point for the generation of the k^{th} PP in a leasing (*resp.* buffering) period, where $k \ge 1$, by $S_{P,L}^{(0)}$ (*resp.* $S_{P,B}^{(0)}$) the total number of transmitting and buffering PPs at the beginning of this period, and by $S_{P,L}^{(k)}$ (*resp.* $S_{P,B}^{(k)}$) the total number of transmitting and buffering PPs seen by the k^{th} PP newly generated in this period. Similar as the leasing period of the SN, a leasing (*resp.* buffering) period of the PN can be divided into multiple disjoint sub-periods $T_{P,L}^{(0)}, T_{P,L}^{(1)}, \ldots$ (*resp.* $T_{P,B}^{(0)}, T_{P,B}^{(1)}, \ldots$), where $T_{P,L}^{(0)}$ (*resp.* $T_{P,B}^{(0)}$) denotes the time interval from the beginning of the leasing period (*resp.* buffering period) to the time point $\tau_{P,L}^{(1)}$ (*resp.* $\tau_{P,B}^{(1)}$) and $T_{P,L}^{(k)} = [\tau_{P,L}^{(k)}, \tau_{P,L}^{(k+1)})$ (*resp.* $T_{P,B}^{(k)} = [\tau_{P,B}^{(k)}, \tau_{P,B}^{(k+1)})$). By the memoryless property of the exponential distribution, we have $E[T_{P,L}^{(k)}] = E[T_{P,B}^{(k)}] = \frac{1}{\lambda_P}$ for each $k \ge 0$.

Recall from Appendix A that the expected number of transmitting and buffering PPs at the beginning of a leasing period is $n_L[i]$, i.e., $E[S_{P,L}^{(0)}] = n_L[i]$, given that the $(n-1)^{\text{st}}$ SP newly generated in the last buffering period sees $i \in [0, n_P]$ transmitting and buffering PPs. Similar as (19), the stationary probability of $S_{P,L}^{(k)}$ for $k \ge 1$ can be derived by the recursive application of the formula: $p(S_{P,L}^{(k)} = x) = \sum_{i=0}^{n_P} \sum_{y=0}^{n_L[i]+k-1} p(S_{P,L}^{(k-1)} = y) p_{P,L}(S_{P,L}^{(k)} = x | S_{P,L}^{(k-1)} = y) p_L(i)$, where $x \ge 0$, $p_L(i) = \frac{p(N,n-1,i)}{\sum_{h=0}^{n_P} p(N,n-1,h)}$ is the probability that the $(n-1)^{\text{st}}$ SP newly generated in the last buffering period will see exactly i transmitting and buffering PPs, and $p_{P,L}(S_{P,L}^{(k)} = x | S_{P,L}^{(k-1)} = y) = p(x|y)$ has appeared in (20) with $\lambda = \lambda_P, \mu = \mu_P$, and $M = N_P$.

Meanwhile, as shown by Appendix A, the expected number of transmitting and buffering PPs at the beginning of a buffering period is $n_B[j]$, i.e., $E[S_{P,B}^{(0)}] = n_B[j]$, given that the number of transmitting and buffering PPs is $j \in [0, n_P]$ at the first time when the SN contains only one transmitting SP in the last leasing period. Thus the stationary probability of $S_{P,B}^{(k)}$ for $k \ge 1$ can be derived by the recursive application of the formula: $p(S_{P,B}^{(k)} = x) = \sum_{j=0}^{n_P} \sum_{y=0}^{n_B[j]+k-1} p(S_{P,B}^{(k-1)} = y) p_{P,B}(S_{P,B}^{(k)} = x | S_{P,B}^{(k-1)} = y) p_B(j)$, where $x \ge 0$, $p_B(j) = \frac{p(N_P,1,j)}{\sum_{h=0}^{n_P} p(N_P,1,h)}$ is the probability that the number of transmitting and buffering PPs is j at the first time when the SN contains only one transmitting SP in the last leasing period, and $p_{P,B}(S_{P,B}^{(k)} = x | S_{S,B}^{(k-1)} = y) = p(x|y)$ has appeared in (20) with $\lambda = \lambda_P, \mu = \mu_P$, and $M = N$.

Because $S_{P,L}^{(k+1)} \le S_{P,L}^{(k)} + 1$ for $k \ge 0$, the PN needs to buffer PPs in each sub-period $T_{P,L}^{(k)}$ of a leasing period only when $S_{P,L}^{(k)} \ge N_P$. Depending on whether $S_{P,L}^{(k+1)} > N_P$, we can derive the expected buffering time of the PN in the sub-period $T_{P,L}^{(k)}$ in a similar way as Appendix B. That is, if $S_{P,L}^{(k)} > N_P$, the expected buffering time incurred by the PN during the sub-period $T_{P,L}^{(k)}$ can be expressed as $E[B_{P,L,1}^{(k)}] = \frac{1}{\lambda_P}[\frac{1}{2}(S_{P,L}^{(k)} + S_{P,L}^{(k+1)}) + 1 - N_P]$; else, it can be expressed as $E[B_{P,L,2}^{(k)}] = \frac{1}{\lambda_P} \sum_{j=1}^{S_{P,L}^{(k)}+1-N_P} \frac{2j-1}{2(S_{P,L}^{(k)}+1-S_{P,L}^{(k+1)})}$. Similarly, if $S_{P,B}^{(k)} > N$, the expected buffering time incurred by the PN during the sub-period $T_{P,B}^{(k)}$ can be expressed as $E[B_{P,B,1}^{(k)}] = \frac{1}{\lambda_P}[\frac{1}{2}(S_{P,B}^{(k)} + S_{P,B}^{(k+1)}) + 1 - N]$; else, it can be expressed as $E[B_{P,B,2}^{(k)}] = \frac{1}{\lambda_P} \sum_{j=1}^{S_{P,B}^{(k)}+1-N} \frac{2j-1}{2(S_{P,B}^{(k)}+1-S_{P,B}^{(k+1)})}$.

By summing up the expected buffering time of the PN in the sub-periods $T_{P,L}^{(0)}, T_{P,L}^{(1)}, \ldots, T_{P,L}^{(n_{P,L})}$, where $n_{P,L} = \lceil \lambda_P E[T_L] \rceil$ is the expected number of PPs newly generated in a leasing period, we can further express the expected buffering time of the PN during a leasing period as:

$$E[B_{P,L}] = \sum_{k=0}^{n_{P,L}} \sum_{S_{P,L}^{(k)}=N_P}^{n_P} p\left(S_{P,L}^{(k)}\right) \left(\sum_{S_{P,L}^{(k+1)}=N_P+1}^{\min\{S_{P,L}^{(k)}+1, n_P\}} E\left[B_{P,L,1}^{(k)}\right] \times p\left(S_{P,L}^{(k+1)}\right) + \sum_{S_{P,L}^{(k+1)}=0}^{N_P} E\left[B_{P,L,2}^{(k)}\right] p\left(S_{P,L}^{(k+1)}\right) \right). \quad (23)$$

Similarly, the expected buffering time of the PN during a buffering period is given by

$$E[B_{P,B}] = \sum_{k=0}^{n_{P,B}} \sum_{S_{P,B}^{(k)}=N}^{n_P} p\left(S_{P,B}^{(k)}\right) \left(\sum_{S_{P,B}^{(k+1)}=N+1}^{\min\{S_{P,B}^{(k)}+1, n_P\}} E\left[B_{P,B,1}^{(k)}\right] \times p\left(S_{P,B}^{(k+1)}\right) + \sum_{S_{P,B}^{(k+1)}=0}^{N} E\left[B_{P,B,2}^{(k)}\right] p\left(S_{P,B}^{(k+1)}\right) \right), \quad (24)$$

where $n_{P,B} = \lceil \lambda_P E[T_B] \rceil$ is the expected number of PPs newly generated in a buffering period.

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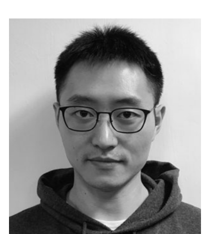

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